Invited Talk@NTUB

Introduction to Property Testing

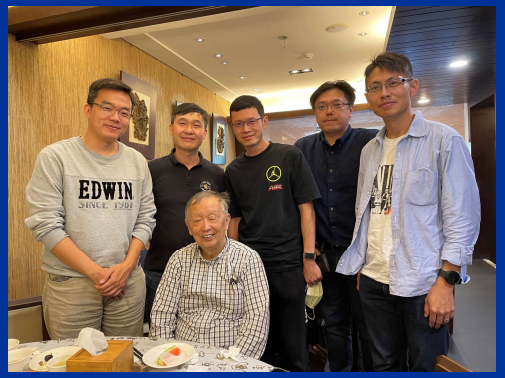
Chuang-Chieh Lin (林莊傑)

Dept. CSIE, Tamkang University

Date: 2 June 2022

Outline

- Introduction
  - Sublinear-time algorithms
  - Notions of approximation
  - Definition of a property tester
- Two simple examples
  - Testing monotonicity of a list
  - Testing connectivity of a graph
- Further readings



About Me (Education)

BS.: Mathematics,

- National Cheng Kung University, 2002.

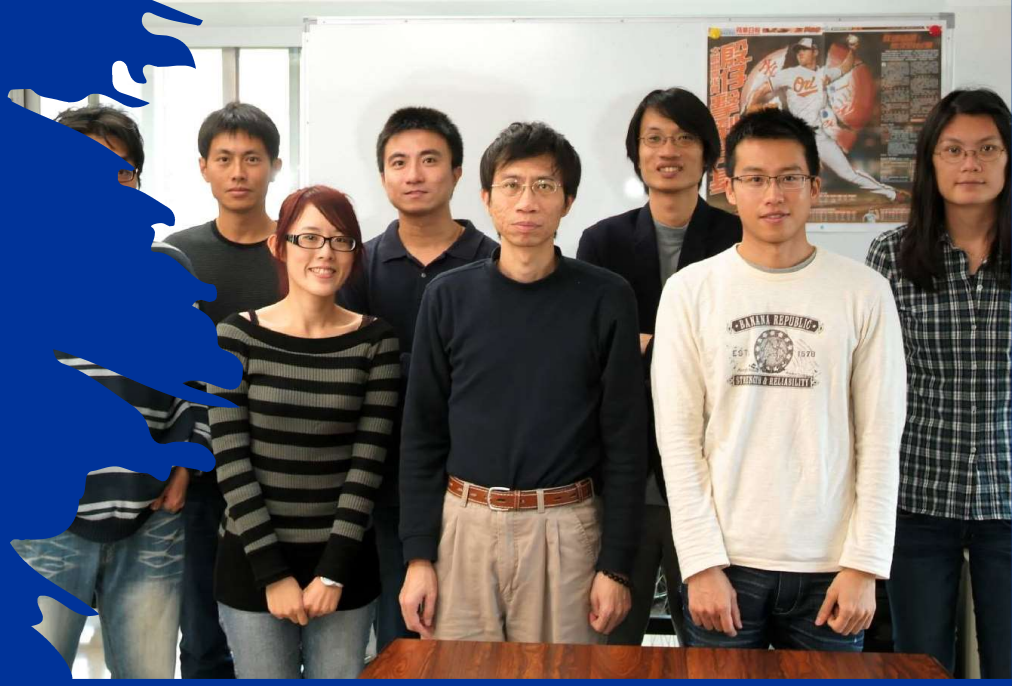
MS.: CSIE,

- National Chi Nan University, 2004.
- Advisor: R.C.T. Lee

Ph.D.: CSIE,

- National Chung Cheng University, 2011.
- Advisors: Maw-Shang Chang & Peter Rossmanith

About Me (Postdoctoral)



@Genomics Research Center, Academia Sinica (alternative military service).

- Bioinformatics, comparative genomics
- Project investigator: Trees-Juen Chuang

2011–2014

2014–2018

@Institute of Information Science, Academia Sinica

- Machine learning, game theory
- Project investigator: Chi-Jen Lu

About Me (Hedge Fund)

Quantitative Analyst (intern) of Point72/Cubist Systematic Strategies

- Hedge Fund; Fintech; Data Science
- US hedge fund (Taipei Branch in 2019)
- CEO & Chairman: Steven A. Cohen

2020–present

2018–2019

Quantitative Analyst of Seth Technologies Inc.

- High-Frequency Trading; Hedge Fund; Fintech; Data Science
- Taiwan based.

About Me (Teaching) – Since 2021 Feburary

What is and Why Property Testing?

Introduction

- With the recent advances in technology, we are faced with the need to process increasingly larger amounts of data in faster times.
- There are practical situations in which the input is so large, that even taking a linear time in its size to provide an answer is too much.
- Making a decision after reading only a small portion of the input, that is, in *sublinear time*, is thus considered to be an very important issue.

Introduction (cont'd)

- Sublinear time algorithms have received a lot of attention recently.

- Recent results have shown that there are optimi-zation problems whose value can be approximated in sublinear time.

Introduction (cont'd)

- However, most algorithms which run in sublinear time must necessarily use randomization and must give an approximate answer.
- Surprisingly though, there are nontrivial problems for which deterministic exact algorithms exist!
- Let us see the following two examples.

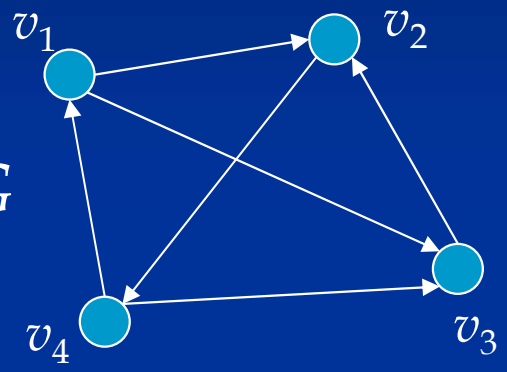
Example 1: Tournament

- A tournament is a digraph such that for each pair of vertices u and v, exactly one of (u, v) and (v, u) is an edge.

- We can interpret the vertices as players such that each pair of players play a match, and an edge from one to another indicates that one player beats another, hence the name tournament.

Tournament (cont'd)

a tournament G



$$M_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

- Assume that we have a tournament G on n vertices represented in adjacency matrix form M_G.
- Thus, the size of G is $\binom{n}{2}$.

Tournament (cont'd)

- **Input:**
  - a tournament G on n vertices represented in adjacency matrix form M_G.
- **Output:**
  - the source of G if it exists, otherwise output "No source exists". (source: the vertex of out-degree $n-1$)

- There exists a deterministic algorithm that finds the source of G (a player who beats all others) if it exists in $O(n)$ time.

Tournament (cont'd)

Algorithm-Source-Finding:

1. $S \leftarrow \{v_1, \ldots, v_n\}$;
2. **while** $|S| > 1$ **do**
   (a) Arbitrary pick $v_i, v_j \in S$;
   (b) **if** $M_G[i, j] = 1$ **then** remove v_j from S;
   else remove v_i from S;
3. Denote the remaining vertex in S by v_r;
4. **For** $i = 1$ to n **do**
   if $M_G[r, i] = 0$ then output "No source exists." and return;
5. Return v_r;

End of the Algorithm

Example 2: Diameter

- Assume that we have n points in a metric space.
- The input is an $n \times n$ distance matrix D such that $D(i, j)$ is the distance between i and j.
- We seek a sublinear time algorithm that outputs $\max_{i,j} D(i, j)$, i.e., the diameter.

Diameter (cont'd)

- **Input:**
  - an $n \times n$ distance matrix D such that $D(i, j)$ is the distance between i and j.
- **Output:**
  - diameter of these n points (i.e., $\max_{i,j} D(i, j)$)

- Consider the following simple algorithm.

Diameter (cont'd)

Algorithm-Diameter:

★ Pick a point u arbitrary and output $z := \max_v D(u, v)$.

End of the Algorithm

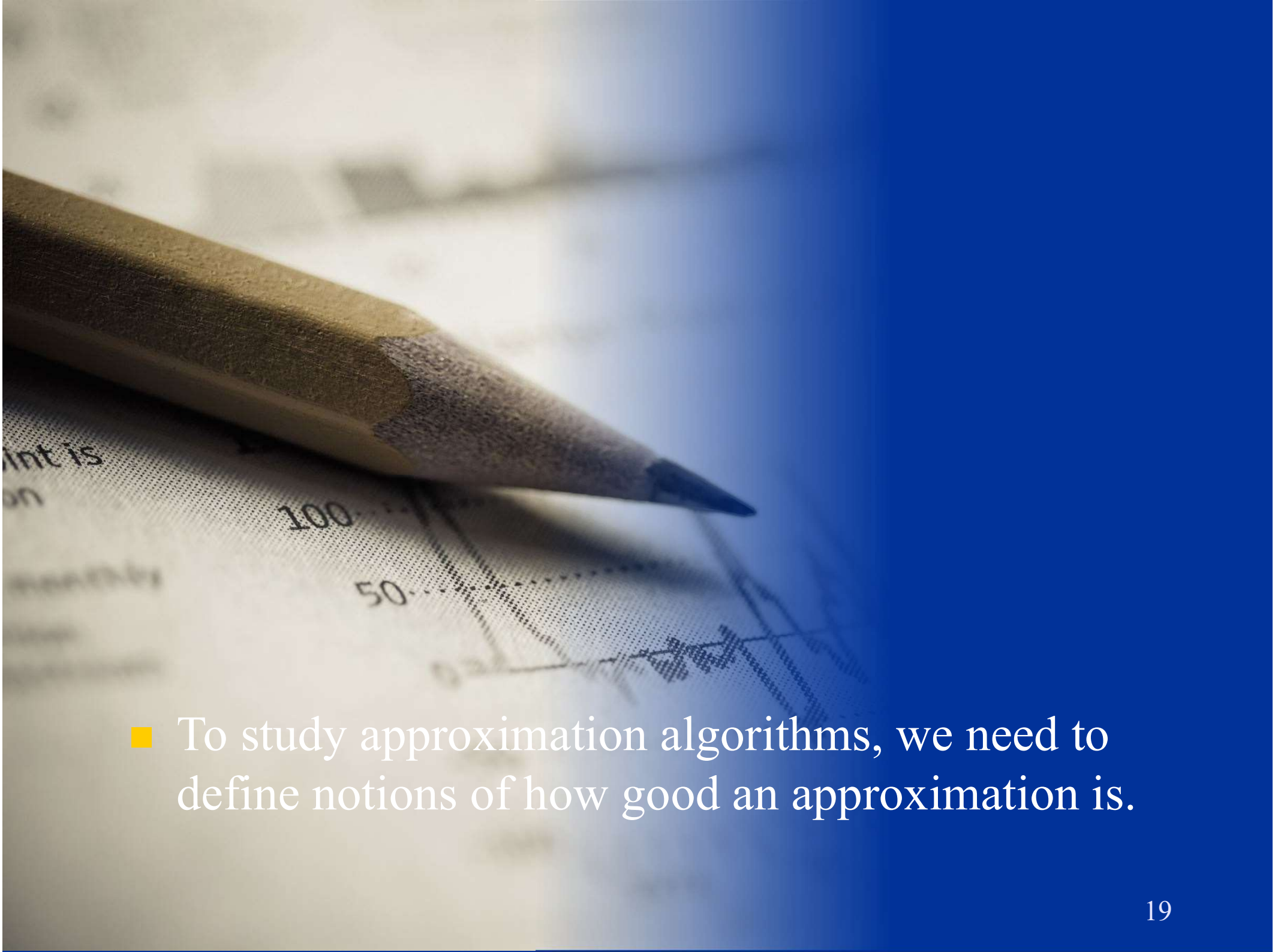
- Clearly this algorithm runs in $O(n)$ time. Moreover, we argue that z, the value returned by this naïve looking algorithm, is a good approximation for the diameter d of the input.

Diameter (cont'd)

- Claim: $d/2 \le z \le d$.
- Proof:
  - Let a and b be two points such that $D(a,b) = d$ and assume that $z = D(u,v)$
  - Since D is a metric space, we have

$$d = D(a,b) \le D(a,u) + D(u,b) \le D(u,v) + D(u,v) = 2z.$$

■



- To study approximation algorithms, we need to define notions of how good an approximation is.

Definitions

Let $\pi(x)$ be the optimal solution of an input x. For $\beta > 1$, we say that A is a *β-multiplicative approximation algorithm* if for all x,

$$\frac{\pi(x)}{\beta} \leq A(x) \leq \beta\pi(x).$$

We say that A is an *α-additive approximation algorithm* if for all x,

$$\pi(x) - \alpha \leq A(x) \leq \pi(x) + \alpha$$

How to approximate a decision problem?

- In addition, *property testing*, an alternative notion of approximation for decision problems, has been applied to give sublinear time algorithms for a wide variety of problems.

- "Still, the study of sublinear time algorithms is very new, and much remains to be understood about their scope." - Ronitt Rubinfeld
  - ACM SIGACT News, Vol. 34, No. 4, 2003.

Property testing

- The notion of property testing was first formulated by Rubinfeld and Sudan.

Ronitt Rubinfeld and Madhu Sudan: Robust charaterization of polynomials with applications to program testing, *SIAM Journal on Computing*, 1996, Vol. 25, pp. 252-271.

Property testing (cont'd)

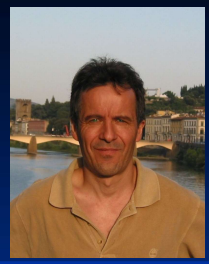
- Due to these two pioneers, plenty results have come out recently.
  - See the "Further readings" for reference.
- Many outstanding scholars have devoted to this topic of research, such as:


Manuel Blum


Madhu Sudan


Ronitt Rubinfeld


Luca Trevisan

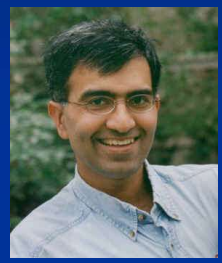

Bernard Chazelle


Noga Alon


Dana Ron


Rajeev Motwani


Oded Goldreich


Sanjeev Arora


Eldar Fischer


Carsten Lund


Tugkan Batu

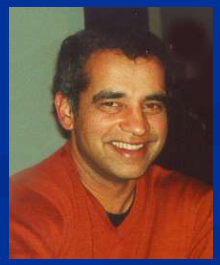

Shafi Goldwasser


Michael Luby


Mario Szegedy


Lance Fortnow

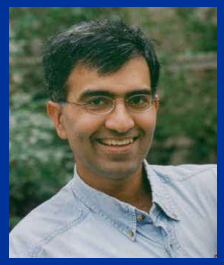

Ravi Kumar


Sampath Kannan


Funda Ergűn

Especially,

- Property testing emerges naturally in the context of program checking and probabilistic checkable proofs (PCP).


Sanjeev Arora


Carsten Lund


Rajeev Motwani


Madhu Sudan


Mario Szegedy

PCP theorem: NP = PCP($O(\log n)$, $O(1)$)

- JACM, Vol. 45, 1998.

Roughly speaking, …

- A property tester is an algorithm which
  - accepts with high probability if the input has a certain property, and
  - rejects with high probability if the input is "far" from the property.
    - ✓ That is, the input *cannot be modified slightly* to make it possess the property.

Property testing (cont'd)

- In order to define a property tester, it is important to define a notion of *distance* from having a property.

- Define a language P to be a class of inputs that have a certain property.
  - For example, *connected* graphs, *monotone increasing* integers, …

Property testing (cont'd)

- Let $\Delta(x, y)$ be the distance function between input x and y, with $\Delta(x, y) \in [0, 1]$ and define

$$d(x, P) = \min_{y \in P} \Delta(x, y)$$

Property testing (cont'd)

- For example, the Hamming distance/ #digits of two 0-1 strings with equal length can be a Δ.

$$\Delta(01001_2, 01110_2) = 3/5.$$

- Let P be a set of 0-1 strings which has fewer 0's than 1's, we can easily have

$$d(01001_2, P) = 1/5.$$

Property testing (cont'd)

- So let us consider the formal definition of a property tester.

Property testing (cont'd)

- A property tester for (P, d) is defined as

  ★ Given input x, $0 < \epsilon < 1$.

  if $x \in P$, then $\mathbf{Pr}[\text{return "Pass"}] \geq 2/3$.
  if $d(x, P) \geq \epsilon$, then $\mathbf{Pr}[\text{return "Fail"}] \geq 2/3$.

  Remark:
  If $d(x, P) \geq \epsilon$, we say x is ϵ-far from P.
  If $d(x, P) \leq \epsilon$, we say x is ϵ-close from P.

The '2/3' can be amplified

- Let's say the tester **A** can achieve:
  - If $x \in P$, then Pr[**A** returns "PASS"] = 1
  - If $d(x, P) \geq \epsilon$, then Pr[**A** returns "FAIL"] = $p \geq \frac{2}{3}$.

- We can run **A** for *m* times and output the majority answer.
  - $m = 3$, the success probability: $\binom{3}{2}\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^2 + \binom{3}{3}\left(\frac{1}{3}\right)^3 \approx 0.74$.
  - $m = 5$, the success probability: $\binom{5}{3}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + \binom{5}{4}\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^4 + \binom{5}{5}\left(\frac{1}{3}\right)^5 \approx 0.79$.
  - $m = 11$, the success probability: ≈ 0.88.
  - $m = 21$, the success probability: ≈ 0.94.
  - $m = 101$, the success probability: ≈ 0.9997.

A simple example

- Consider the following example to figure out the concept of property testing.

- Suppose we have a sequence of n numbers, $x_1, \ldots, x_n$, we would like to determine if the sequence is monotonically increasing.
  - Input: $x_1, \ldots, x_n$
  - Output: Accepts or Rejects.

Testing monotonicity of a list

- Any deterministic decision algorithm runs in $\Omega(n)$ time to read the input and make a decision.
- On the other hand, a property testing algorithm exists such that it
  - accepts, if the sequence is monotonically increasing
  - rejects with probability greater than 2/3, if more than εn of the x_i need to be removed so that the resulting sequence becomes monotonically increasing.

Testing monotonicity of a list (cont'd)

- WLOG, we can assume that all x_i's are distinct.
  - Since we can interpret x_i as (x_i, i), which breaks ties without changing order.

- Consider the following simple approach which can not be ensured to run in sublinear time.

Testing monotonicity of a list (cont'd)

Algorithm 1

★ Select i randomly and test whether $x_i < x_{i+1}$. Then return "Pass" if $x_i < x_{i+1}$, and return "Fail" otherwise.

- Consider the following sequence which is very far from monotonically increasing:

PASS

Testing monotonicity of a list (cont'd)

- Generally, such sequence $x_1, x_2, \ldots, x_n$ can be written as the following form:

  $m, 2m, \ldots, km,$

  $m-1, 2m-1, \ldots, km-1, \ldots,$

  $1, m+1, 2m+1, \ldots, (k-1)m+1.$ (thus $n = mk$)

  where m, k are two integers greater than 1.

- For example, when $m = 4$, $k = 3$:

  4, 8, 12, 3, 7, 11, 2, 6, 10, 1, 5, 9

Testing monotonicity of a list (cont'd)

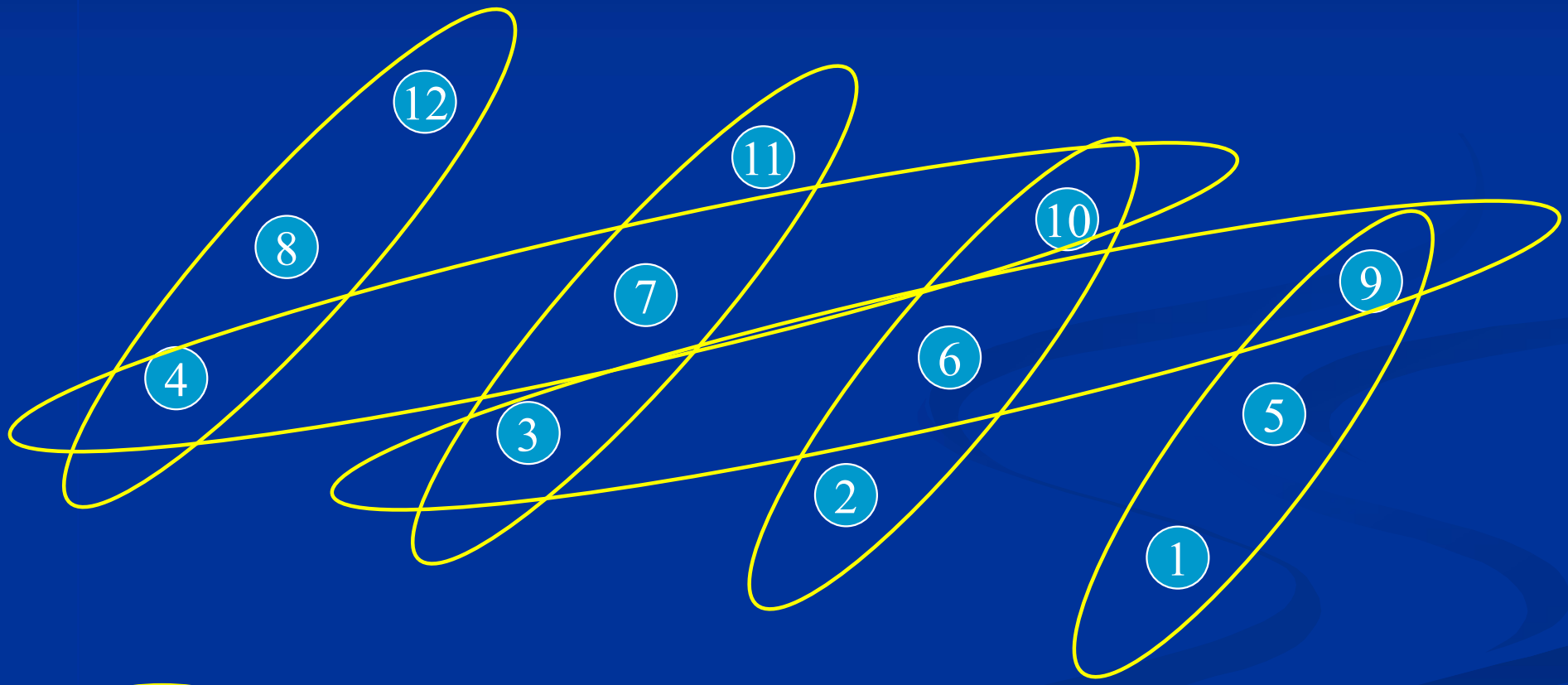
- The distance of such sequence from monotonically increasing is at least ½.
  - WHY?
  - For example,

    2, 4, 1, 3 → 2, 4 or 2, 3 or 1, 3

    for monotonically increasing

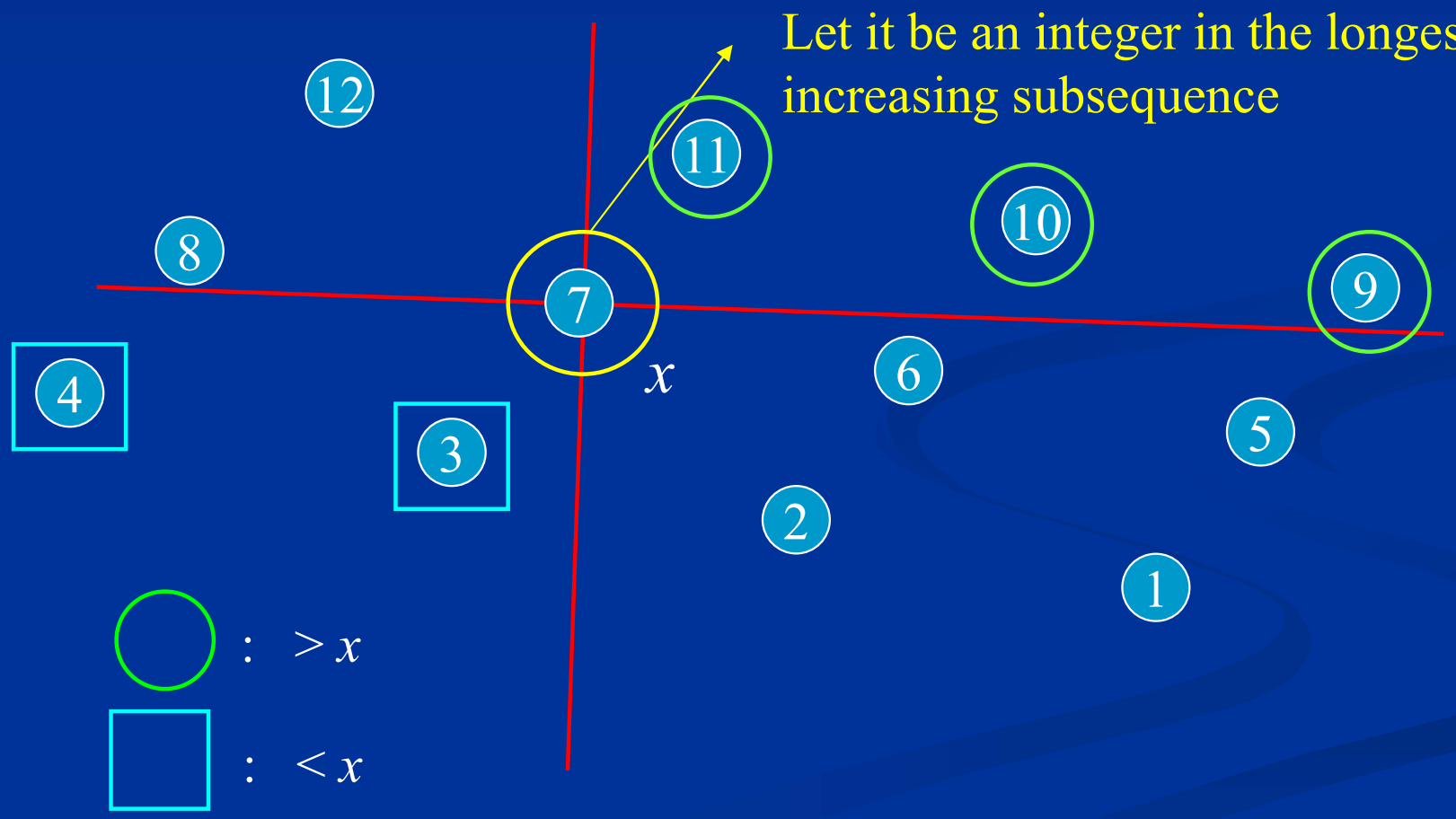
Testing monotonicity of a list (cont'd)

- See the following illustration: ($m = 4$, $k = 3$)



Testing monotonicity of a list (cont'd)

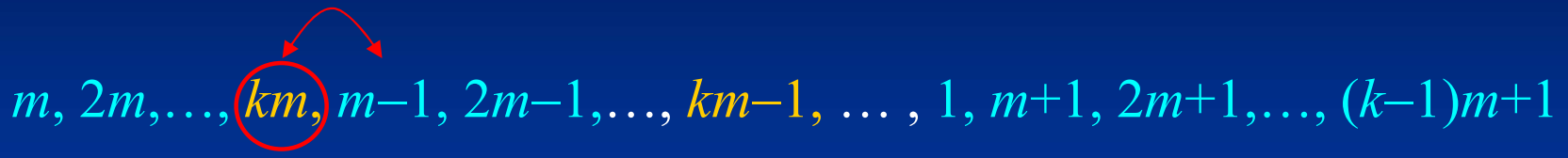
- See the following illustration: ($m = 4$, $k = 3$)



Testing monotonicity of a list (cont'd)

- We can easily prove that the length of a longest monotonically increasing subsequence in such a sequence must be at most k,
  - Exercise. (Hint: Consult the previous illustration.)
- So the distance of such sequence from monotonically increasing is at least $n - k = (m-1)k$, which is at least ½ of the length of the sequence.
  - For example, 2, 4, 1, 3 → 2, 4 or 2, 3 or 1, 3

Testing monotonicity of a list (cont'd)



- Algorithm 1 does not detect that the sequence is not monotonically increasing as long as it does not query a pair of locations of a yellow integer and its next integer respectively.
- Thus Algorithm 1 will need $\Omega(k)$ queries, that is, repeatedly runs $\Omega(k)$ times.
  - WHY?

Testing monotonicity of a list (cont'd)

$m, 2m, \ldots, km, m-1, 2m-1, \ldots, km-1, \ldots, 1, m+1, 2m+1, \ldots, (k-1)m+1$

- The probability that Algorithm 1 doesn't query any yellow integer is larger than $1 - 1/k$ for each run.
- The probability that Algorithm 1 queries a yellow integer at least once during $c \cdot k$ runs is less than $1 - (1-1/k)^{ck}$.

Testing monotonicity of a list (cont'd)

- $1 - (1-1/k)^{ck} \searrow 1 - 1/e^c > 2/3$ when k is large and $c > 1$.
  - That is, if we don't run Algorithm 1 for more than $\Omega(k)$ times, Algorithm 1 will not query any yellow integer with high probability (when k is large and $c > 1$.)
- However, we cannot ensure the probability that Algorithm 1 query a yellow integer at least once during $c \cdot k$ runs is at least 2/3.

Testing monotonicity of a list (cont'd)

- Thus, the time complexity of this algorithm cannot be ensured to be sublinear.
- Try another one!

Testing monotonicity of a list (cont'd)

- Consider another algorithm, which is a little sophisticated.

Algorithm 2

★ Samples the sequence at random points and checks if these random points form a monotonically increasing sequence.

★ Return "Pass" if they do, and return "Fail" otherwise.

Testing monotonicity of a list (cont'd)

- However, consider the following sequence, which is again very far from monotonically increasing.

$$m, m-1, \ldots, 1, 2m, 2m-1, \ldots, m+1, 3m, \ldots, 2m+1, \ldots$$

- Again, the distance of this sequence from monotonically increasing is at least ½.
- The algorithm detects that this sequence is not monotonically increasing only if two of its query points fall within $[km, (k-1)m+1]$ for some k.

Testing monotonicity of a list (cont'd)

$m, m-1, \ldots, 1, \quad 2m, 2m-1, \ldots, m+1, \quad 3m, \ldots, 2m+1, \ldots$

- However, by the Birthday Paradox, this is unlikely if m is a constant and the number of samples is $o((n/m)^{1/2}) = o(n^{1/2})$.
- With high probability, the values of the query points will form a monotonically increasing sub-sequence.
- Thus Algorithm 2 does not work well.

- Can we do better?
  - YES!





F. Ergün, S. Kannan, R. Kumar, R. Rubinfeld and M. Viswanathan proposed a $O((1/\varepsilon) \log n)$ property tester.

- JCSS, Vol. 60, 2000

Testing monotonicity of a list (cont'd)

- Consider the following algorithm. [EKKRV00]

Algorithm 3$((x_1, \ldots, x_n), \epsilon)$

★ Repeat Step 1 to 3 for $O(1/\epsilon)$ times:

1. Pick i uniformly at random from 1 through n.
2. Query x_i.
3. Perform binary search for x_i. If the search does not found x_i, return "Fail" (i.e., Reject).

★ Return "Pass" (i.e., Accept) if all searches are successful.

For example,

```
Begin binary search
```

index	1	2	3	4	5	6	7
value	21	9	1	3	5	8	17

Search for value **1**.

Output: **Fail!**

Another example,

```
Begin binary search
```

index	1	2	3	4	5	6	7
value	21	9	1	3	5	8	17

Search for value 8.

Output: **Pass!**

Testing monotonicity of a list (cont'd)

- Algorithm 3 runs in time $O((1/\varepsilon)\log n)$ since each binary search takes $O(\log n)$ time.
- If the sequence $\{x_i\}$ is monotonically increasing, then clearly the algorithm accepts.
- We need to show that if at least εn of the sequence need to be removed for it to be monotonically increasing, then the algorithm rejects (resp. accepts) with probability at least 2/3 (resp., less than 1/3).
  - Suppose not, that Algorithm 3 accepts with probability at least 1/3.

Testing monotonicity of a list (cont'd)

- We call index i is
  - "*good*" if the binary search for x_i is successful,
  - "*bad*" otherwise.

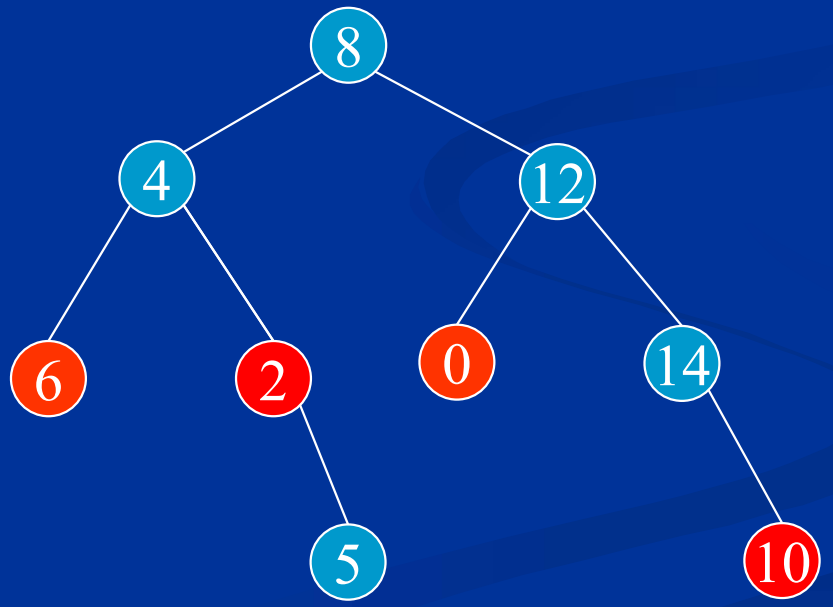
Testing monotonicity of a list (cont'd)

- For example,

index	1	2	3	4	5	6	7	8	9
value	6	4	2	5	8	0	12	14	10

: good ones

: bad ones



Testing monotonicity of a list (cont'd)

- We claim that less than εn of the indices are bad.
  - Otherwise, each time through the loop, the algorithm finds a bad index with probability at least ε.
  - Then Algorithm 3 accepts with probability at most $(1-\varepsilon)^{c/\varepsilon} < e^{-c} < 1/3$ for some constant c.
  - A contradiction then occurs.
- Now, the remaining part is to prove that the good points indeed form a monotonically increasing subsequence.

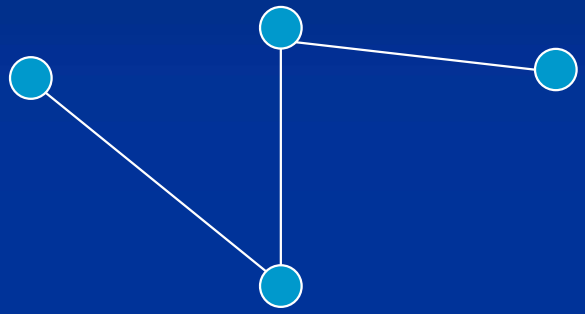
Testing monotonicity of a list (cont'd)

- Consider any two good indices i, j , where $i < j$.
- Consider the first point in the binary search path where x_i and x_j diverge and assume that point has value u.
- Since i and j are good and $i < j$, we can conclude that $x_i \leq u \leq x_j$. This concludes the proof. ■

- Now, let us consider another problem:
  Testing connectivity of a graph.

Connected and Disconnected

connected



disconnected

Degree bound

- We say a graph $G(V, E)$ has a degree bound d if for each vertex $v \in V$,

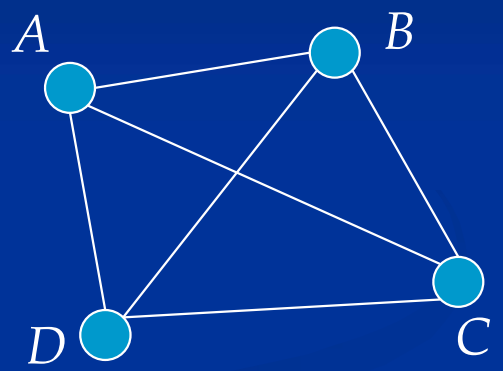
$$\deg(v) \leq d$$

where $\deg(v)$ is the number of vertices adjacent to v in G.

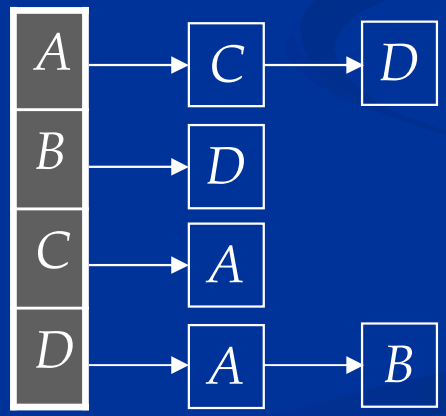
Graph representations

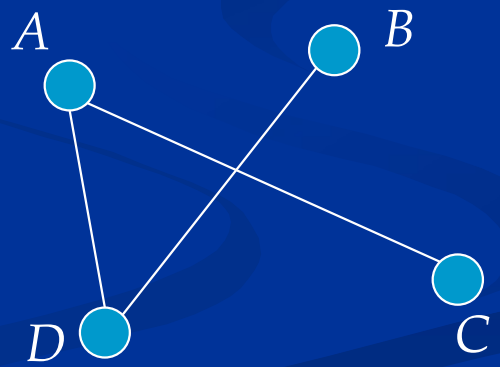
- Adjacency matrix
  - For dense graphs

	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0



- Adjacency list
  - For sparse graphs





Testing connectivity of a graph

- We will adopt the adjacency list model with a given degree bound d to proceed with our discussion.
  - The graph possesses $O(dn)$ edges.

Testing connectivity of a graph (cont'd)

- ★ **Input:** a graph $G(V, E)$ with bounded degree d , given as adjacency list
- ★ **Desired property:** P = a class of connected graphs with bounded degree d

Let $\mathcal{G}_n^d$ denote the set of graphs of n nodes with a bounded degree d.

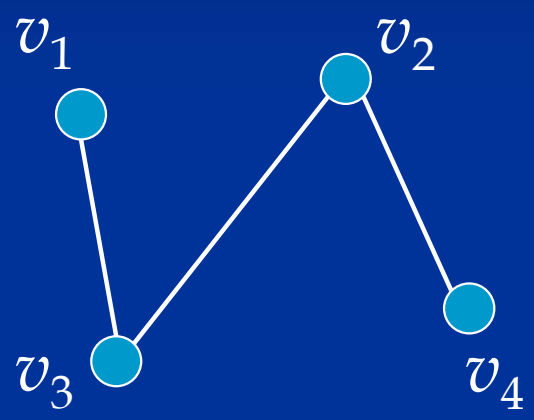
Testing connectivity of a graph (cont'd)

- Let $G \in \mathcal{G}_n^d$, we define the distance of G from connected to be

$$\text{dist}(G, P) = \frac{2\rho_d(G)}{dn}$$

where $\rho_d(G)$ is the minimum number of modifications of edges needed for G to be connected such that the degree bound d is still maintained.

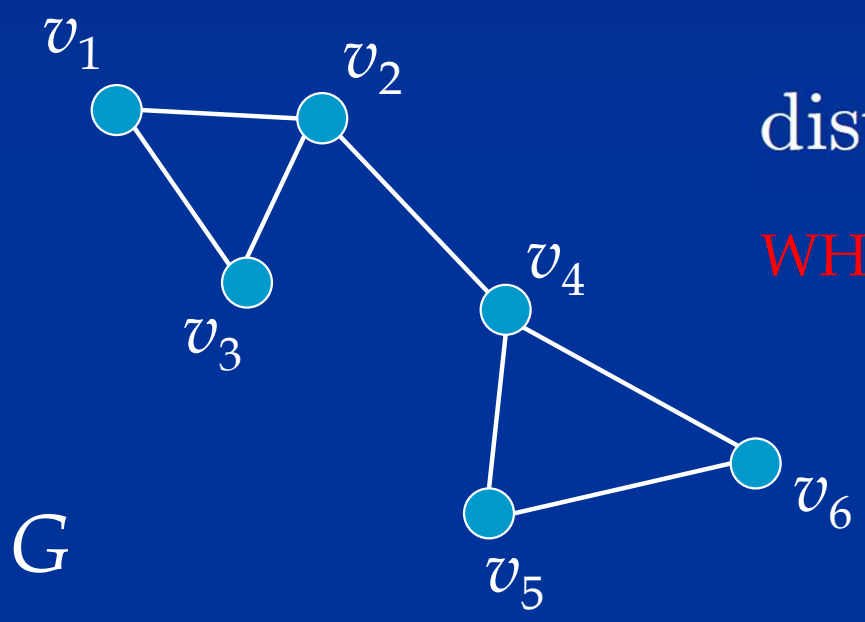
For example, ($d = 2$)



G

$$\text{dist}(G, P) = \frac{2\rho_2(G)}{dn} = \frac{1}{4}.$$

Another example, ($d = 2$)



$$\text{dist}(G, P) = \frac{2\rho_2(G)}{dn} = \frac{1}{2}.$$

WHY?

Idea

- If a graph is far from connected, there must be many components,
  - That in turn implies that there are many small components.
- Consider the following algorithm proposed by O. Goldreich and D. Ron.


- Algorithmica, Vol. 32, 2002.

Testing connectivity of a graph (cont'd)

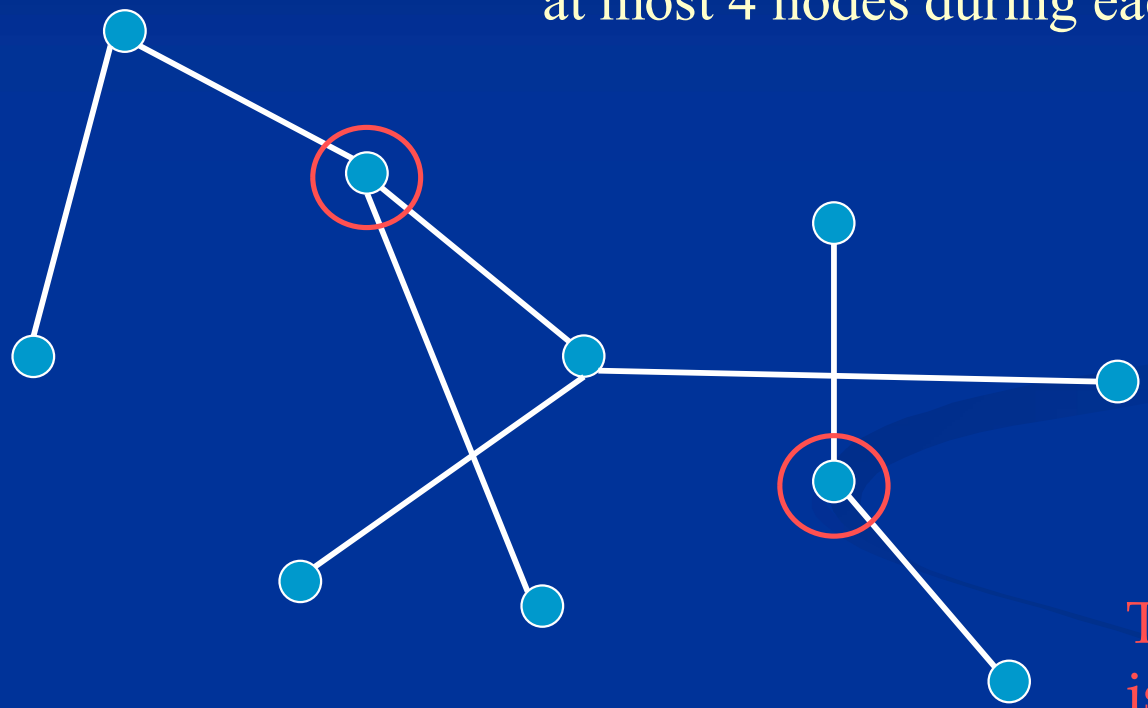
Algorithm GR(G, ϵ) [GR02]

1. Pick $m = O(\frac{1}{\epsilon d})$ nodes of G uniformly at random.
   Let S denote the set of these picked nodes.
2. **For** each node $s \in S$, **do** BFS and stop **if**:
   (a) $\frac{8}{\epsilon d}$ nodes have been reached
   (b) exhaust the component
3. **If** (b) ever happens, return "Fail";
   otherwise, return "Pass".

(Here we assume that $|V| \geq 8/\epsilon d$.)

An illustration

Pick 2 nodes of the graph, and traverse at most 4 nodes during each BFS.



STOP

The component is exhausted!

Halt and output: "Fail"

Testing connectivity of a graph (cont'd)

- The running time of Algorithm GR is

$$O(\frac{1}{\epsilon d} \cdot \frac{8}{\epsilon d} \cdot d) = O(\frac{1}{\epsilon^2 d}),$$

  which is sublinear.

- Why does this algorithm work?

Testing connectivity of a graph (cont'd)

- For $G \in \mathcal{G}_n^d$, if $G \in P$, it is obvious that the algorithm must output "Pass".
  - Maybe you don't think that this is trivial. You can prove this claim for an easy exercise.

- So, what if $G \notin P$?
  - We have to prove that if G is far from P, (i.e., G is far from connected with degree bound d) Algorithm GR will output "Fail" with probability at least 2/3.

Testing connectivity of a graph (cont'd)

- Consider the following observation first.

- Observation:

  If $G \in \mathcal{G}_n^d$ is ϵ-far from connected, then G has at least $\epsilon dn/2$ connected components.

- Proof:
  - If G has less than $\varepsilon dn/2$ connected components, we can add less than $\varepsilon dn/2$ edges to make G connected.
  - G is not ε-far from connected. (Because $\varepsilon dn / dn = \varepsilon$) ■

Testing connectivity of a graph (cont'd)

- Lemma 1:

A class of connected graphs with bounded degree *d*

If $G \in \mathcal{G}_n^d$ is ϵ-far from P, then G has at least $\epsilon dn/4$ connected components.

- Proof: Exercise!
  - Hint: Consider the previous observation and the second example for illustrating $\text{dist}(G, P)$.

Testing connectivity of a graph (cont'd)

- Corollary 1:

  If $G \in \mathcal{G}_n^d$ is ϵ-far from P, then G has at least $\epsilon dn/8$ connected components each containing less than $\frac{8}{\epsilon d}$ nodes.

- Proof:
  - Let $n_<$ be the number of components of size less than $\frac{8}{\epsilon d}$. → We call them small components for simplicity.
  - Let $n_>$ be the number of components of size at least $\frac{8}{\epsilon d}$.

Testing connectivity of a graph (cont'd)

- Assume that G is ε-far from P. Then from Lemma 1 we have that G has at least $\varepsilon dn/4$ connected components.
- Since $n_< + n_>$ is the total number of connected components in G, we have $n_< + n_> \geq \varepsilon dn/4$.
- Since $n_> \cdot 8/\varepsilon d \leq n$, we have $n_> \leq \varepsilon dn/8$.
- Therefore, $n_< \geq \varepsilon dn/4 - \varepsilon dn/8 = \varepsilon dn/8$, the corollary immediately follows. ■

Testing connectivity of a graph (cont'd)

- Theorem 1:

  Let $G \in \mathcal{G}_n^d$

  - ★ if G is connected, then Algorithm GR return "Pass"
  - ★ if G is ϵ-far from P, then $\mathbf{Pr}[\text{Algorithm GR return "Fail"}] \geq \frac{2}{3}$.

- Proof of Theorem 1 is as follows.

Testing connectivity of a graph (cont'd)

- If G is connected, Algorithm GR must output "Pass".
  - Trivial.

- Consider the case that G is ε-far from P.

Testing connectivity of a graph (cont'd)

- By Corollary 1,

$$\begin{aligned}
&\mathbf{Pr}[s \text{ is in a small component}] \\
&= \frac{\text{number of nodes in small components}}{n} \\
&\geq \frac{\text{number of small components}}{n} \\
&\geq \frac{\epsilon d}{8}.
\end{aligned}$$

From Corollary 1.

Each component is of size at least one and they are disjoint.

Testing connectivity of a graph (cont'd)

- Since m is chosen to be $c/\varepsilon d$ for some constant c, we have

$$\begin{aligned}
& \mathbf{Pr}[\text{no } s \text{ is in a small component}] \\
\leq\ & (1-\frac{\epsilon d}{8})^{\frac{c}{\epsilon d}} \\
\leq\ & e^{-c'} \\
<\ & \frac{1}{3}.
\end{aligned}$$

These inequalities holds as long as we pick c large enough (c' is a constant that depends on c).

Therefore, the proof is done. ■

I think I should end this talk now.

Textbook

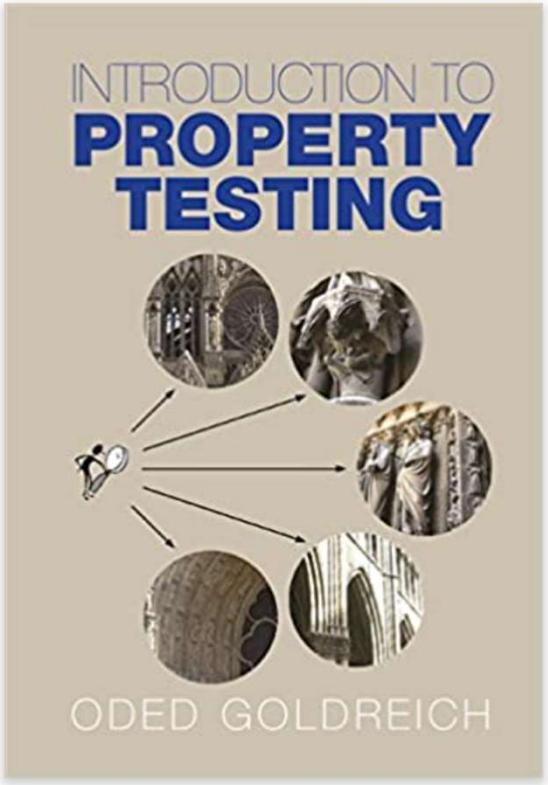
Books › Engineering & Transportation › Engineering

Introduction to Property Testing 1st Edition

by Oded Goldreich (Author)

★★★★★ 1 rating

Look inside



ISBN-13: 978-1107194052
ISBN-10: 9781107194052
Why is ISBN important?

Kindle	Hardcover
$78.00	$74.45

Buy new:
$74.45

Usually ships within 2 to 3 days.

Ships from and sold by Amazon.com.

Available at a lower price from other sellers tha

More Buying Choices
8 New from $74.45 | 3 Used from $71.93

Further readings

- A Brief Introduction to Property Testing by Oded Goldreich:
  - https://www.wisdom.weizmann.ac.il/~oded/COL/pt-intro.pdf
- Sublinear Time Algorithms by Ronitt Rubinfeld:
  - http://theory.lcs.mit.edu/%7Eronitt/sublinear.html

Thank you.